## CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/06
Paper 6 (Extended)
May/June 2009
1 hour 30 minutes

Additional Materials: Answer Booklet/Paper Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
You may use a pencil for any diagrams or graphs.

Answer both parts $\mathbf{A}$ and $\mathbf{B}$.

You must show all relevant working to gain full marks for correct methods, including sketches, even if your answer is incorrect.

In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.

At the end of the examination, fasten all your work securely together.
The total number of the marks for this paper is 40 .

Answer both parts A and B.
A. INVESTIGATION

REMOVING DISCS
24 marks
(10)
(1)
(2)
(9)

(8)


(6)

Ten discs, numbered 1 to 10 , form a circle.
You remove the disc numbered 1, and, going clockwise, leave the next one, remove the one after that, leave the next one, and so on until only one disc remains.
The discs which you remove are, in order, numbered $1,3,5,7,9,2,6,10,8$.
The remaining disc is numbered 4 .
1 Copy and complete the table showing the number on the remaining disc when you have 2 discs, 3 discs, 4 discs, ...., 20 discs in the circle.

| Number of discs in the circle | Number on the remaining disc |
| :---: | :---: |
| 2 | 2 |
| 3 | 2 |
| 4 | 4 |
| 5 |  |
| 6 |  |
| 7 | 8 |
| 8 | 4 |
| 9 | 6 |
| 10 | 8 |
| 11 | 10 |
| 12 | 14 |
| 13 | 16 |
| 14 |  |
| 15 | 6 |
| 16 | 8 |
| 17 |  |
| 18 |  |
| 19 |  |
| 20 |  |

2 When you have 2, 4, 8 or 16 discs in the circle, the number on the remaining disc is always the same as the total number of discs in the circle. Assume this pattern continues.

Write down the next three numbers when this happens.

3 Use question 2, and any patterns you see in your table, to find the number on the remaining disc when the circle contains
(a) 65 discs,
(b) 125 discs,
(c) 200 discs,
(d) 100000 discs.

4 Find an algebraic expression for the number of discs in the circle, when the remaining disc is numbered 10 .

5 Consider the original ten discs again.
Remove the disc numbered 10, and going anticlockwise, leave the next one, remove the one after that, leave the next one, and so on until only one disc remains.
You should finish with the disc numbered 7 .
(a) (i) Write down the numbers on the discs in the order in which you remove them.
(ii) When you started with the disc numbered 1 and worked clockwise, the order was

$$
\begin{array}{lllllllll}
1 & 3 & 5 & 7 & 9 & 2 & 6 & 10 & 8
\end{array}
$$

Compare this order with the order you have written down in part (a) (i).
Explain how the two orders are related.
(b) (i) When you have $n$ discs in the circle and work clockwise from the disc numbered 1 , the number on the remaining disc is $x$.
When you work anticlockwise from the disc numbered $n$, the number on the remaining disc is $y$. Find a rule connecting $x, y$ and $n$.
(ii) Find the number on the remaining disc when there are 100 discs and you work anticlockwise by first removing the disc numbered 100 .

Part $B$ is on the next page.


The diagram shows two vertical poles $(A P$ and $B Q)$ that support the ends $(A$ and $B)$ of a hanging chain. The chain is symmetrical about the $y$-axis.
The poles are each 7.52 metres high and are 4 metres apart
The lowest point of the chain is 2 metres above the horizontal ground $(P Q)$.
The units are in metres.

1 Write down the co-ordinates of $A, B$ and the lowest point of the chain.
2 The scientist Galileo chose one of the following models for the shape of a hanging chain.
Which model best fits the hanging chain in the diagram?

$$
\begin{gathered}
y=a x+b \\
y=|a x+b| \\
y=a x^{2}+b \\
y=a \sin x+b
\end{gathered}
$$

3 Use the co-ordinates of the lowest point of the chain to find the value of $b$ in your chosen model.
4 Find the value of $a$ in your chosen model.
5 According to your model, how high is the chain above a point on the ground that is 50 cm from one of the poles?
6 Today, engineers use the model $y=k\left(w^{x}+\frac{1}{w^{x}}\right), w>1$, for a hanging chain.
(a) Use the co-ordinates of the lowest point of the chain to find the value of $k$.
(b) Find the value of $w$ for this model.

7 The two models give heights for the hanging chain. The difference in these heights is $h$.
(a) Sketch the graph of $h$ against $x$.
(b) Find the greatest difference in height between the two models.

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